2^{nd} Order Linear Prediction Applied to Digital Frequency Estimation for Different Disturbances

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***Abstract*—To maintain the quality of energy supply and the correct operation of equipments, the electrical frequency is a parameter of high importance in Electric Power Systems. This work presents a digital frequency estimation technique based on network voltage waveforms analysis that are decomposed into their α and β components using the Clarke Transform. Second order linear prediction is used in order to predict the future values of these components. In addition, a logic to reduce the prediction error is applied to improve the estimation response. The network frequency is then estimated as a function of the angle resulting from the multiplication between the complex signal and the signal given by the prediction of α and β components. The technique were tested for signals with ramped, exponential and damped sinusoidal frequency variations, as well as in the presence of White Gaussian Noise, and evaluated in terms of convergence time, minimum and maximum errors before and after convergence, showing that this innovative method has great precision and robustness in different simulation situations.**

***Index Terms*—digital frequency estimation, protection of electrical power systems, Clarke's transform, linear prediction, performance indices.**

I. Introduction

The digital estimation of electrical frequency is of great importance for Electric Power Systems (EPSs) since the equipment can be damaged when the frequency variation limits are exceeded, impairing the supply and quality of electrical energy. So, the actuation of protection and control devices are quite important for sudden variations in the fundamental frequency, which can lead the system to operational restrictions, in order to maintain the acceptable frequency levels for the continuous energy supply.

In EPS, the fundamental frequency is related to the quality of electrical energy, where, ideally, this has constant values of frequency and effective voltage. In steady state, frequency oscillations are allowed, but with small amplitudes. Frequencies outside the limitations imposed for each system can indicate the occurrence of faults or overloads as mentioned in [1].

Thus, damage to equipment connected to the electrical network, such as generators, transformers, motors, capacitor banks and transmission lines, are directly related to frequency variations, and affect the stability of the system [2].

In these circumstances, it is necessary to accurately estimate the electrical frequency, to mitigate damage to the system through protection actuation, and to save the quality of the supplied energy, through control and operational actions. In this context, several methodologies have been presented in the specialized literature to estimate the electrical frequency, such as those proposed by [1] to [16]. However, no method proposed so far is able to guarantee accuracy in frequency estimation in all operational conditions.

This work presents a new digital frequency estimation method, which is based on the prediction of the α and β components of the network voltage signals, using their linear predictions of 2^{nd} order. The proposed technique was tested for cases involving ramped, exponential and damped sinusoidal frequency variations, as well as signals corrupted by White Gaussian Noise (WGN), presenting great precision and robustness for frequency estimation, reaching the convergence criterion in all analyzed cases.

II. Proposed Technique for Frequency Estimation

To analyze the performance of the Frequency Estimation method via Linear Prediction of Clarke Components (LPCC), the results were evaluated in terms of performance indices, such as convergence time, convergence interval, minimum, and maximum errors before and after convergence.

The LPCC is based on obtaining the Clarke Components of three-phase voltage signals, through the $\alpha\beta$ Transform, according to (1), where n represents the actual sample.

$$\begin{bmatrix} v_\alpha(k) \\ v_\beta(k) \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} \tag{1}$$

The resulting complex signal is given by (2).

$$u(k) = v_\alpha(k) + j v_\beta(k) \tag{2}$$

The method proposed here uses linear prediction to estimate the future values of the α and β signals.

Thus, according to [18], the linear prediction model recursively represents the time series of signal samples over a time interval, as follows in (3).

$$v(k) = c_1 \cdot v(k-1) + \cdots + c_i \cdot V(k-i) \tag{3}$$

Where $k-1$, ..., $k-i$ indicate past samples of the v signal. Also, c_1, ..., c_i are the linear prediction coefficients, i is the order of the model. In this study the 2^{nd} order model was used as follows.

$$\begin{bmatrix} v_\alpha(k) \\ v_\alpha(k-1) \end{bmatrix} = \begin{bmatrix} v_\alpha(k-1) & v_\alpha(k-2) \\ v_\alpha(k-2) & v_\alpha(k-3) \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e(k) \\ e(k-1) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} v_\beta(k) \\ v_\beta(k-1) \end{bmatrix} = \begin{bmatrix} v_\beta(k-1) & v_\beta(k-2) \\ v_\beta(k-2) & v_\beta(k-3) \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e(k) \\ e(k-1) \end{bmatrix} \tag{5}$$

Resulting in Equation (6).

$$v(k+1) = c_1 \cdot v(k) + c_2 \cdot v(k-1) + c_3 \cdot v(k-2) + c_4 \cdot v(k-3) \tag{6}$$

Where $v(k+1)$ indicates the future estimated value of v_α or v_β. After obtaining the future values of the v_α and v_β through the linear prediction, these are assigned to the value of the estimated complex signal (u_{est}), according to (7).

$$u_{est}(k+1) = v_{\alpha_{est}}(k+1) + jv_{\beta_{est}}(k+1) \tag{7}$$

With the values $u(k)$ and $u_{est}(k+1)$, γ(k) is calculated according to Equation (8).

$$\gamma(k) = u_{est}(k+1) \cdot u(k)^* \tag{8}$$

In (8), $u(k)^*$ represents the complex conjugate of $u(k)$. Finally, the frequency estimation of the system is given according to (9).

$$f_{est}(k) = \frac{f_s}{2\pi} \cdot \tan^{-1}\left\{\frac{Im[\gamma(k)]}{Re[\gamma(k)]}\right\} \tag{9}$$

In (9), *Re* and *Im* represent, respectively, the real and imaginary parts of γ(k) and f_s is the sampling frequency. The flowchart of this methodology can be seen in Figure 1.

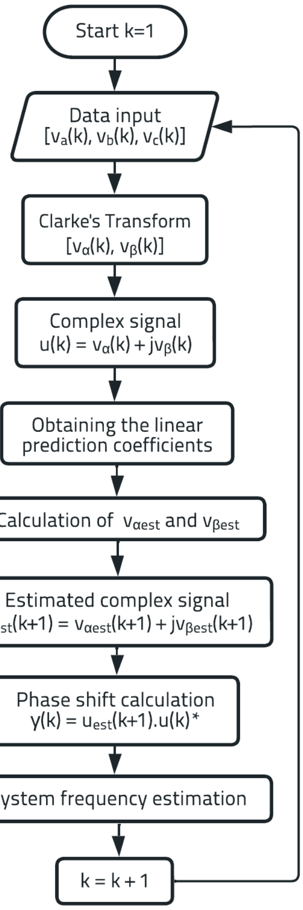

Fig. 1. Flowchart of the Frequency Estimation method via Linear Prediction of Clarke Components (LPCC).

III. Performance Indices

In order to validate the quality of the frequency estimation of the proposed methodology (LPCC), the following performance indices were analyzed: convergence time, convergence interval, maximum error before convergence, minimum error before convergence, maximum error after convergence, and minimum error after convergence.

A. Convergence Time (CT)

The Convergence Time is the instant in which the estimation absolute error is less than 0.05 Hz during 3 cycles after the first estimation that fulfils the convergence condition. When an estimation error is less than 0.05 Hz, the algorithm saves the respective instant and counts the number of estimations that fulfil the convergence criterion. If this count reaches a number equivalent to 3 cycles of samples, the saved instant is considered the convergence instant. If the absolute error of the estimation is greater than 0.05 Hz before the counter reaches three cycles, the saved value is deleted and a new instant will be searched if the convergence condition is satisfied again. Furthermore, if the absolute error does not meet the established criterion, the method response is considered non-convergent.

B. Convergence Interval (CI)

The Convergence Interval is the interval between the Convergence Time (CT) and the frequency Variation Instant (VI), as indicated in (10).

$$CI = CT - VI \tag{10}$$

For protection purposes, it is desirable that the frequency estimators response has the shortest possible convergence interval. Thus, the smaller the CI, the better the frequency estimator performance according to [4].

C. Absolute Error

It is given by the difference between the estimated frequency (f_{est}) for the actual sample (k) and the reference frequency for the same instant (f_{ref}), according to (11).

$$error(k) = |f_{est}(k) - f_{ref}(k)| \tag{11}$$

From the absolute error, the maximum and minimum values of the frequency estimator response absolute errors are determined, before and after their respective convergence, determining the following performance indices: Maximum Absolute Error Before Convergence (AEBCmax), Minimum Absolute Error Before Convergence (AEBCmin), Maximum Absolute Error After Convergence (AEACmax) and Minimum Absolute Error After Convergence (AEACmin). To prevent an improper operation of the frequency relay during a frequency transient, it is desirable that AEBC to be as small as possible according to [8]. Furthermore, it is desirable that the frequency estimation error be as small as possible during the entire signal processing, since this parameter indicates the accuracy of the method. Lower estimation errors indicates better performance.

IV. Results

The proposed frequency estimator was tested for computationally generated signals, which simulate different frequency variations. In all cases, the fundamental frequency was assumed as 60 Hz, being this the initial frequency. The amplitude of all signals was defined as 1 p.u. Also, for ramped or exponentially frequency variations the deviation begins at 0.5 second, while for the cases with damped sinusoidal frequency variation it begins at 0.2 second. All cases were simulated for 16, 32, 64, 96, 128 and 256 samples per cycle.

The Figures present the estimated frequency for the sampling rate of 15,360 Hz (256 samples/cycle at 60 Hz), while the Tables in the Appendix present the performance indices for all analyzed sampling rates. It is noteworthy that for all sampling rates the results were satisfactory.

All the voltage signals were generated according to (12) to (14), where $f(k)$ represents the frequency signal which has a particular equation for each type of deviation [17], as presented in the following sections.

$$v_A(k) = A \cdot \cos[2\pi \cdot f(k) \cdot t(k)] \quad (12)$$

$$v_B(k) = A \cdot \cos\left[2\pi \cdot f(k) \cdot t(k) - \left(\frac{2\pi}{3}\right)\right] \quad (13)$$

$$v_C(k) = A \cdot \cos\left[2\pi \cdot f(k) \cdot t(k) + \left(\frac{2\pi}{3}\right)\right] \quad (14)$$

A. Signals with Ramped Frequency Variation

For this type of variation, frequency signal deviation is given by (15).

$$f(k) = f_0 + \Delta f \cdot t(k) \quad (15)$$

In (15), f_0 represents the initial frequency. Fig. 2 shows the good performance of the proposed method for a signal with ramped frequency variation from 60 to 65 Hz. Table I, in the Appendix, shows the performance indices obtained by sampling rates from 16 to 256 samples/cycle, where the best results are highlighted in green and the worst in red. In this case, 256 samples/cycle presented the best results for CT and CI performance indices. The best values for AEBCmax and AEBCmin were found with 16 to 128 samples/cycle while the worst were obtained by 256 samples/cycle. For AEBCmax and AEBCmin, the best results were found using 256 and 96 samples/cycle, respectively, while the worst results were obtained with 32 and 16 samples/cycle.

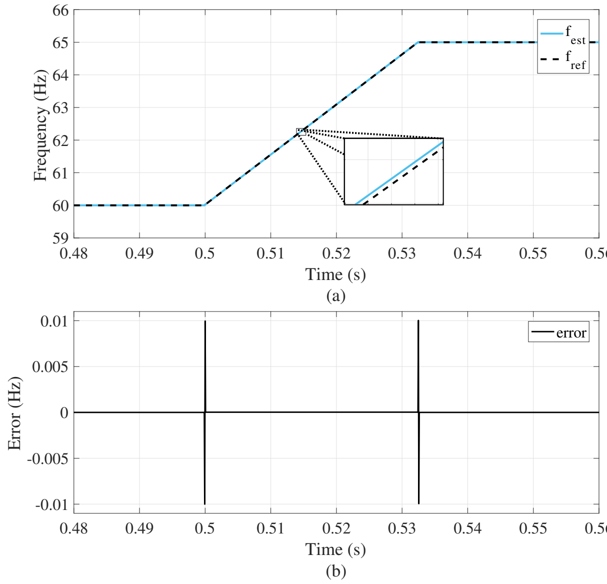

Fig. 2. (a) Performance of 2^{nd} order linear prediction frequency estimation for ramped frequency variation. (b) Estimation error.

B. Signals with Exponentially Frequency Variation

For this type of variation, in (12) to (14) the frequency exponentially varies according to (16) from 60 Hz to higher or lower values.

$$f(k) = f_0 + \Delta f \cdot \left(1 - e^{-\frac{t(k)}{\tau}}\right) \quad (16)$$

In (16), f_0 represents the initial frequency, Δf is the frequency variation amplitude and τ is the time constant of the exponential function. Signals were generated for different values of Δf and τ.

Fig. 3 presents the response for a signal with exponential frequency variation generated for Δf = -5 Hz and $\tau = 0.05$ s using the proposed technique. The performance indices obtained from each sample rate analyzed are presented in Table II of the Appendix.

In this case, the sampling rate of 256 samples/cycle presented the best results for the CT, CI and AEBCmax indices, while for the AEBCmin indices the best results were presented using 16 to 128 samples/cycle. For AEACmax and AEACmin the best results were obtained with 256 samples/cycle.

C. Signals with Damped Sinusoidal Frequency Variation

These signals were generated using to (12) to (14), where the frequency during the transient was given by (17).

$$f(k) = f_0 + \Delta f \sin \cdot \left(\omega_f \cdot t(k) \cdot e^{-\frac{t(k)}{\tau}}\right) \quad (17)$$

In (17), f_0 represents the initial frequency, Δf is the amplitude of the change, ω_f is the angular frequency of the frequency change, and τ is the time constant of the exponential function.

Fig. 4 presents the results for the estimated frequency using linear prediction for the processing of voltage signals with damped frequency variation from 60 Hz to 55 Hz, generated with Δf= 5 Hz, ω_f = 10π, and τ= 0.15 s.

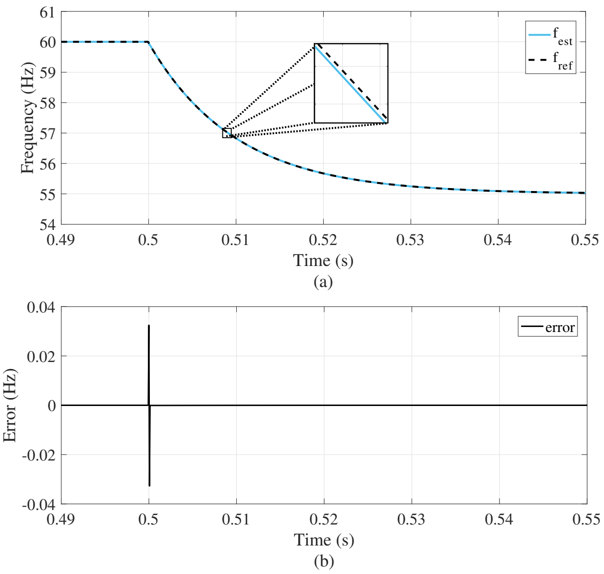

Fig. 3. (a) Performance of 2^{nd} order linear prediction frequency estimation for exponentially frequency variation. (b) Estimation error.

For this case, the best values of the CT, CI and AEBCmax indices were verified using 256 samples/cycle, while the worst was found with 16 and 32 samples/cycle. For the AEBCmin, the best response was verified using 16 samples/cycle and the worst with 128 samples/cycle. For the AEACmax and AEACmin indices, the sampling of 256 samples/cycle had the best result, while the worst was verified using 16 samples/cycle. The performance indices obtained with each analyzed sampling rates are presented in Table III of the Appendix.

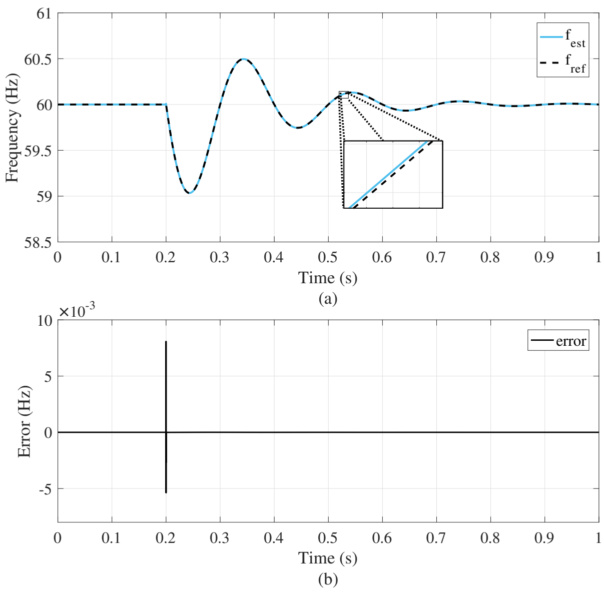

Fig. 4. (a) Performance of 2^{nd} order linear prediction frequency estimation for damped frequency variation. (b) Estimation error.

D. Signals Corrupted by WGN

The method performance was tested with the presence of WGN in the voltage signals. An average filter with one cycle moving window was used for a signal corrupted by 40 dB Signal to Noise Ratio (SNR). For this case, the results are shown in Fig. 5 using a sampling rate of 256 samples/cycle.

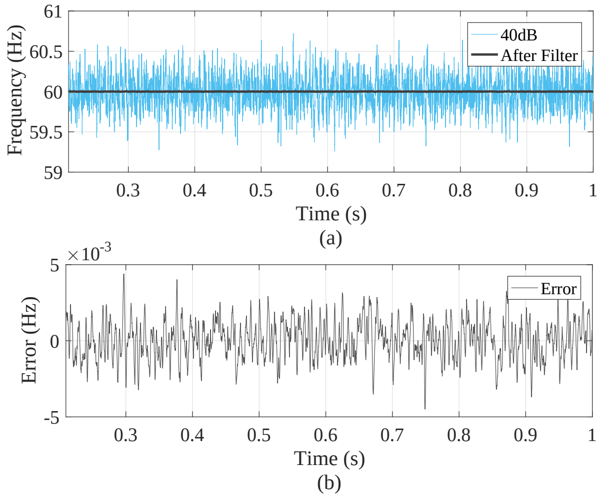

Fig. 5. (a) Frequency estimation using linear prediction for White Gaussian Noise of 40dB in the voltage signals, and the frequency estimation after the filtering. (b) Estimation error.

In this case, the best responses were observed for the rates of 96 to 128 samples/cycle. However, for 16 samples/cycle CT, CI and AEBCmax best responses were verified among all the sampling rates tested. The better value of AEBCmin (8.64601E-07 Hz) was obtained using 128 samples/cycle. For AEACmax, the best performance was verified at 256 samples/cycle, wich reaches the value of 4.49924E-03 Hz. Best AEACmin was obtained with 96 samples/cycle (1.18444E-08 Hz). The performance indices obtained by each analyzed sampling rates are presented in Table IV of the Appendix.

V. Conclusions

This work presented a new method applicable to digital relays, which aims to estimate the electrical frequency on EPS. Such technique was computationally evaluated in order to analyze its performance indices in different situations in which the electrical power system is susceptible.

Finally, it was concluded that the new method has an excellent performance, presenting acceptable values of performance indices even for the cases where the signals are corrupted by WGN. So, the proposed new technique can be used as a powerful tool for frequency estimation task.

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VI. Appendix

TABLE I
PERFORMANCE INDICES FOR A RAMP FREQUENCY VARIATION (60 HZ TO 65 HZ).

Samples/Cycle	CT	CI	AEBCmax	AEBCmin	AEACmax	AEACmin
16	5.51042E-01	5.10417E-02	1.00000E-02	0.00000E+00	4.48250E-03	4.18264E-03
32	5.50521E-01	5.05208E-02	1.00000E-02	0.00000E+00	1.16243E-02	6.57963E-12
64	5.50260E-01	5.02604E-02	1.00000E-02	0.00000E+00	1.04409E-02	2.85638E-12
96	5.50174E-01	5.01736E-02	1.00000E-02	7.10543E-15	1.01988E-02	0.00000E+00
128	5.50130E-01	5.01302E-02	1.00000E-02	0.00000E+00	1.01125E-02	7.57439E-12
256	5.50065E-01	5.00651E-02	1.00283E-02	2.70717E-12	8.43272E-11	7.33280E-11

TABLE II
PERFORMANCE INDICES FOR AN EXPONENTIAL FREQUENCY VARIATION (60 HZ TO 55 HZ, Δf =-5 HZ AND τ = 0.05 S).

Samples/Cycle	CT	CI	AEBCmax	AEBCmin	AEACmax	AEACmin
16	5.73958E-01	7.39583E-02	4.94625E-01	0.00000E+00	1.54240E-04	1.56319E-13
32	5.54688E-01	5.46875E-02	2.53751E-01	0.00000E+00	1.47219E-04	2.13163E-14
64	5.50521E-01	5.05208E-02	1.28528E-01	0.00000E+00	2.77049E-05	8.52651E-14
96	5.50347E-01	5.03472E-02	8.60564E-02	7.10543E-15	8.25993E-06	3.55271E-14
128	5.50260E-01	5.02604E-02	6.47596E-02	0.00000E+00	3.48985E-06	2.84217E-14
256	5.50130E-01	5.01302E-02	3.27072E-02	5.11591E-12	4.36519E-07	0.00000E+00

TABLE III
PERFORMANCE INDICES FOR A DAMPED FREQUENCY VARIATION (60 HZ TO 55 HZ, Δf =5 HZ, ω_f = 10π AND $\tau = 0.15s$).

Samples/Cycle	CT	CI	AEBCmax	AEBCmin	AEACmax	AEACmin
16	2.51042E-01	5.10417E-02	1.04855E-01	0.00000E+00	1.00414E-02	1.95602E-06
32	2.51042E-01	5.10417E-02	6.16483E-02	5.31486E-12	1.69310E-03	9.21894E-08
64	2.50260E-01	5.02604E-02	3.19975E-02	2.50608E-11	2.26461E-04	3.54979E-09
96	2.50174E-01	5.01736E-02	2.14693E-02	3.60103E-11	6.79382E-05	1.22306E-10
128	2.50130E-01	5.01302E-02	1.61353E-02	9.24771E-11	2.87829E-05	2.09099E-10
256	2.50065E-01	5.00651E-02	8.08321E-03	9.12692E-11	3.61272E-06	8.41993E-12

TABLE IV
PERFORMANCE INDICES FOR WHITE GAUSSIAN NOISE IN THE FREQUENCY.

Samples/Cycle	CT	CI	AEBCmax	AEBCmin	AEACmax	AEACmin
16	3.67708E-01	3.67708E-01	3.72712E-02	1.56678E-04	3.71901E-02	2.99770E-05
32	3.68229E-01	3.68229E-01	1.80397E-01	1.39790E-04	1.91593E-02	4.17666E-07
64	3.68750E-01	3.68750E-01	2.97006E-01	3.94093E-06	8.64615E-03	2.58621E-06
96	3.68924E-01	3.68924E-01	3.44610E-01	9.79302E-07	7.42733E-03	1.18444E-08
128	3.69010E-01	3.69010E-01	3.73025E-01	8.64601E-07	7.43668E-03	1.08747E-06
256	3.69206E-01	3.69206E-01	4.23059E-01	1.57916E-06	4.49924E-03	8.91703E-08